MODELLING OF TERRAIN FOR NECESSITIES OF MILITARY OBJECTS MOVEMENT SIMULATION

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A problem of terrain information gaining for objects movement simulation is considered. A mathematical model of terrain based on information from topographical map is presented. This model is used to constructing of trafficable routes network (TRN). Two kinds of TRN, rough and precise, are considered. The first type of the network is based on the real roads in the terrain only. The precise network is constructed basing on partition of the terrain into squares of topographical homogeneous areas. A coding of this information is done by utilization of quadtrees. Examples of creating of TRN are shown.

Introduction

In many practical military problems which are solved by utilization of computer technology, as one of more elements of warfare simulation models, movement simulation of military objects is considered. Objects movement is an essential element of combat actions (direct or during its preparation) and is strictly related to one of kind of military activities which is known as maneouvre (in meaning of movement or redeployment). To make possible computer simulation of objects movement in battlefield, this battlefield must be adequately described. Such description allows for computer processing and image of terrain information. This paper deals with the problem of terrain information gaining for objects movement simulation and it is important to show how we can present topographical information to use it during movement simulation or redeployment planning.

This paper is organized as follows.
In section **1** a mathematical terrain model based on information from the topographical map has been created. Section **2** contains the methods of constructing two types of trafficable routes networks (TRN). Two models of TRN has been presented : rough and precise. For the second type of TRN, algorithms of constructing these networks have been described. Additionally, examples of utilization of the method have been shown. **Summary** contains resume of the presented models and some general notices on this subject.

1. A mathematical terrain model (MTM)

In order to gain and process terrain information we must present it in the digital form. Such a notation may be the mathematical terrain model (MTM). This formal description of terrain is easy to convert to a numerical form and can be stored in any database structure.

MTM may be defined as a set of numerical data which allow for gaining, by means of specified algorithms, of information about the terrain area shape, position and covering on the definite zone [3]. As the data may be used: coordinates x, y, z of individual terrain points; data which describe a form of terrain lines, e.g. contour lines, terrain profile, etc.; data which describe rivers, roads, forests, railway lines, etc. These data may be acquired by means of terrain measurements, fotogrametric reports, digitization of contour lines and others objects on the map, satellite data.

In many cases we can use a modern digitizers to obtain a precise information from the map. In order to receive terrain information we must partition a sheet of the map into the *n* rows and *m* columns with Δx and Δy steps. The left lower corner of the map sheet is a start point. When we know this point and steps Δx and Δy we can describe a position of the other points on the map with Δx and Δy precision. For each point the vector of parameters must be given. The number of vector coordinates depends on terrain object the considered point belongs to. In our case vectors should be described very precisely because this description will be used to create trafficable routes networks for numerical terrain presentation. We must model different characters of the map: the point characters (points present objects as: trees, houses, wells, etc.), linear objects like: roads, railways, rivers, canals, drains and area objects as: forests, lakes, seas, swamps. We use the following descriptions to model terrain information:

R_{pkt} - number of types of point objects;
R_{pow} - number of types of area objects;
R_{lin} - number of types of linear objects;
L^i_{pkt} - number of the i-th type of point objects, $i=\overline{1,R_{pkt}}$;
L^i_{pow} - number of the i-th type of area objects, $i=\overline{1,R_{pow}}$;
L^i_{lin} - number of the i-th type of linear objects, $i=\overline{1,R_{lin}}$;
$O^{i,j}_{pkt}$ - the j-th point object of the i-th type, $j=\overline{1,L^i_{pkt}}$, $i=\overline{1,R_{pkt}}$;
$O^{i,j}_{pow}$ - the j-th area object of the i-th type, $j=\overline{1,L^i_{pow}}$, $i=\overline{1,R_{pow}}$;
$O^{i,j}_{lin}$ - the j-th linear object of the i-th type, $j=\overline{1,L^i_{lin}}$, $i=\overline{1,R_{lin}}$;
WSP - set of well-ordered triples describing coordinates of point in the system of coordinates which has been used in digitization:

$$WSP=\left\{\langle x,y,z\rangle : x\in R^+\cup\{0\}, y\in R^+\cup\{0\}, z\in R\right\}$$

where :

x, y - coordinates of the map point estimated with the reference system used in digitization;
z - height of terrain point;

We use as a reference system coordinates of the left lower corner of map sheet.

1.1 Modelling of point objects

Point objects may be described as follows:

$$O^{i,j}_{pkt}=w\in WSP \tag{1}$$

For objects of this type we don't need a precise characteristic (e.g. height of tree, depth of well, etc.) because this information is needless from the point of view of its appropriation (during movement or redeployment).

1.2 Modelling of area objects

Generally, the area object (in principle its contour line) may be presented in the following way:

$$O^{i,j}_{pow} = \left\langle S^{i,j}_{pow}, A^{i,j}_{pow} \right\rangle \ , \ i = \overline{1, R_{pow}}, \ j = \overline{1, L^{i}_{pow}} \tag{2}$$

where :

$$S^{i,j}_{pow} = \left\langle G^{i,j}_{pow}, FW^{i,j}_{pow}, FU^{i,j}_{pow} \right\rangle; \tag{3}$$

$G^{i,j}_{pow}$ - Berge's graph, strongly connected, without loops, with nodes of the contour line of the area object as graph vertices while arcs of a graph link neighbouring vertices. In this graph for each vertex there exists one simple cycle exactly (which begins and ends in the selected vertex and has various arcs and vertices) which is a Hamiltonian and Eulerian cycle simultaneuously.

$$G^{i,j}_{pow} = \left\langle W^{i,j}_{pow}, U^{i,j}_{pow}, P^{i,j}_{pow} \right\rangle \tag{4}$$

$W^{i,j}_{pow}$ - set of vertices of the area object;

$U^{i,j}_{pow}$ - set of arcs of the area object;

$P^{i,j}_{pow}$ - relation , $P^{i,j}_{pow} \subset W^{i,j}_{pow} \times U^{i,j}_{pow} \times W^{i,j}_{pow}$;

$FU^{i,j}_{pow}$ - function defined on arcs of the graph of the j-th area i-th type object;

$FW^{i,j}_{pow}$ - function defined on vertices of the graph of the j-th area i-th type object;

$A^{i,j}_{pow}$ - attributes vector of the j-th area i-th type object,

$$A^{i,j}_{pow} = \left\langle a^{i,j,n}_{pow} \right\rangle_{n=\overline{1,N^{i}_{pow}}} \tag{5}$$

$a^{i,j,n}_{pow}$ - the n-th attribute of the j-th area i-th type object;

N^{i}_{pow} - the number of attributes of the i-th type area object;

The appropriate way of selecting contour nodes is an important problem. These should be particular points on the contour as points of bends, points of contour curvature, etc. The more points we select the better we reconstruct the contour shape [10], [11]. It is important for various calculations which may be done by using above information (e.g. checking whether a selected point lies inside of the contour (inside of the area object), etc. [4]).

For MTM the following kinds of area objects have been specified [5], [7]:

1) lakes and other water reservoirs (artificial and natural);
2) forests, orchards, trees and bushes groups;
3) swamps and wet meadows;
4) mounds, coals;
5) excavations, ravines, canyons;
6) soil homogeneous area;
7) buildings (occupying some area in the scale of the map and not being point objects);

We will present the attribute vector and function described on the graph's arcs for the first kind of area object. Attributes of other kinds of area objects may be constructed by analogy.
For lakes (1st kind of area object) we have :

$$A^{1,j}_{pow} = \left\langle a^{1,j,1}_{pow}, a^{1,j,2}_{pow} \right\rangle \in R^{+} \times R_{grunt} \tag{6}$$

$$FU^{1,j}_{pow} : U^{1,j}_{pow} \rightarrow R_{brzeg} \tag{7}$$

$$FW^{1,j}_{pow} : W^{1,j}_{pow} \rightarrow WSP \tag{8}$$

where:

$a_{pow}^{1,j,1}$ - depth of water reservoir;

$a_{pow}^{1,j,2}$ - kind of the bottom;

R_{brzeg} - set of indices of possible kinds of reservoir watersides;

R_{grunt} - set of indices of possible kinds of soils (kinds of bottom too).

1.3 Modelling of linear objects

Generally, a model of the linear object may be the following :

$$O_{lin}^{i,j} = \left\langle S_{lin}^{i,j}, A_{lin}^{i,j} \right\rangle \quad , \ i = \overline{1, R_{lin}}, \ j = \overline{1, L_{lin}^{i}} \tag{9}$$

where :

$$S_{lin}^{i,j} = \left\langle G_{lin}^{i,j}, FW_{lin}^{i,j}, \xi^{i,j} \right\rangle;$$

$G_{lin}^{i,j}$ - graph without loops, where points of object contour bend (as crossings, roads corners, other particular points) and characteristic points neighbouring with this object (as the beginning and the end of the ditch, etc.) create vertices of this graph,

$$G_{lin}^{i,j} = \left\langle W_{lin}^{i,j}, U_{lin}^{i,j}, P_{lin}^{i,j} \right\rangle \tag{10}$$

$W_{lin}^{i,j}$ - set of graph vertices of linear object;

$U_{lin}^{i,j}$ - set of graph arcs of linear object;

$P_{lin}^{i,j}$ - relation, $P_{lin}^{i,j} \subset W_{lin}^{i,j} \times U_{lin}^{i,j} \times W_{lin}^{i,j}$;

$\xi^{i,j}$ - set of functions described on the graph arcs for the j-th linear object of the i-th type, $\xi^{i,j} = \left\{ \xi_1^{i,j}, \xi_2^{i,j}, \ldots, \xi_{g_{lin}^{i,j}}^{i,j} \right\}$;

$g_{lin}^{i,j}$ - number of functions described on the graph arcs for the j-th linear object of the i-th type;

$FW_{lin}^{i,j}$ - function described on the graph vertices;

$A_{lin}^{i,j}$ - attributes vector of the j-th linear object of the i-th type,

$$A_{lin}^{i,j} = \left\langle a_{lin}^{i,j,n} \right\rangle_{n=\overline{1,N_{lin}^{i}}} \quad , \tag{11}$$

$a_{lin}^{i,j,n}$ - the n-th attribute of the j-th linear object of the i-th type;

N_{lin}^{i} - number of attributes of the i-th type linear object;

For MTM the following kinds of linear objects which influence possibility of traffic have been specified [5], [7]:

1) roads, cuttings;
2) ditches, tidal waves, dikes;
3) bridges, viaducts;
4) contour lines;
5) walls, enclosures;
6) rivers, canals;

We will show the attribute vector and functions described on the graph arcs and vertices for the first kind of linear objects only. Attributes of other kinds of these objects may be constructed by analogy.

We define the set A_{lin}^{i} of linear object attributes vectors of the i-th type :

$$A_{lin}^{i} = \left\{ A_{lin}^{i,j} \right\}_{j=\overline{1,L_{lin}^{i}}} \tag{12}$$

For roads (the 1st kind of linear object) we have :

$$A_{lin}^{1,j} = \left\langle a_{lin}^{1,j,1}, a_{lin}^{1,j,2}, a_{lin}^{1,j,3} \right\rangle \in R_{drogi} \times (R^{+})^{2} \tag{13}$$

where :

$a_{lin}^{1,j,1}$ - kind of road material;

$a_{lin}^{1,j,2}$ - width of road (without shoulders);

$a_{lin}^{1,j,3}$ - width of road (with shoulders);

R_{drogi} - set of indices of possible road materials.

We define the function:

$$F1_{lin}^{1,j} : W_{lin}^{1,j} \rightarrow R_{wezla} \tag{14}$$

where:

R_{wezla} - set of indices of possible kinds of nodes for linear object of the “road” type.

Moreover, we describe the set RN_{wezla} of names of vertices types for “road” type linear object and F_{naz} function following :

RN_{wezla}={crossing, the beginning of a ditch (mound), the end of a ditch (mound), road corner, other point of a road} ;

$$F_{naz} : R_{wezla} \xrightarrow{1-1} RN_{wezla} ; \tag{15}$$

Element of the RN_{wezla} set - „other point of a road” may describes, for example the beginning and the end of the road narrowing, etc. Description of F_{naz} function is presented in the Table 1.

Tab. 1 Description of F_{naz} function

nr∈R_{wezla}	1	2	3	4	5
F_{naz}(nr)	crossing	the beginning of a ditch (mound)	the end of a ditch (mound)	road corner	other point of a road

$$FW_{lin}^{1,j} : W_{lin}^{1,j} \rightarrow WSP \tag{16}$$

$$\xi^{1,j} = \left\{ \xi_{1}^{1,j}, \xi_{2}^{1,j}, \xi_{3}^{1,j} \right\} \tag{17}$$

$$\xi_{1}^{1,j} : U_{lin}^{1,j} \rightarrow \left\{ \left\langle a_{1}^{1,j}, a_{2}^{1,j} \right\rangle : a_{1}^{1,j} \in R,\ a_{2}^{1,j} \in R^{+} \cup \{0\} \right\} \tag{18}$$

$$\xi_{2}^{1,j} : U_{lin}^{1,j} \rightarrow A_{lin}^{1} \tag{19}$$

$$\xi_{3}^{1,j} : U_{lin}^{1,j} \rightarrow \left\{ \left\langle a_{1}^{1,j}, a_{2}^{1,j} \right\rangle : a_{1}^{1,j} \in R,\ a_{2}^{1,j} \in R^{+} \cup \{0\} \right\} \tag{20}$$

where:

$\xi_{1}^{1,j}$ - function which describe the left side of the road;

$\xi_{2}^{1,j}$ - function which describe the road;

$\xi_{3}^{1,j}$ - function which describe the right side of the road;

$a_{1}^{1,j}$ - depth of the ditch (height of the mound);

$a_2^{1,j}$ - width of the ditch (mound);

Notice

The values $a_1^{1,j}=0$ i $a_2^{1,j}=0$ describe that no ditch (mound) exists. Negative values of $a_1^{1,j}$ desribe depth of the ditch.

2. Models of trafficable routes networks (TRN)

We will consider two type of trafficable routes networks : rough and precise. Rough TRN base on real roads only which exist in terrain. Precise TRN takes possibility of traffic not only on the roads. Additionally, during creation these networks we take into account information about effects of weather and the season of the year on the traffic possibilities.

2.1 Model of the rough TRN

Model of the rough network is the following :

$$S_{zgr} = \langle G_{zgr}, \psi_{zgr}, \zeta_{zgr} \rangle \tag{21}$$

where:

G_{zgr} - Berge graph, which the roads system is described on;

ψ_{zgr} - set of functions described on the graph's vertices;

ζ_{zgr} - set of functions described on the graph's arcs;

We now define the elements of S_{zgr} :

$$G_{zgr} = \langle W_{zgr}, U_{zgr}, P_{zgr} \rangle \tag{22}$$

where:

W_{zgr} - set of graph's vertices;

U_{zgr} - set of graph's arcs;

P_{zgr} - relation , $P_{zgr} \subset W_{zgr} \times U_{zgr} \times W_{zgr}$;

W_{zgr} satisfies the condition :

$$W_{zgr} = \left(W_{zgr}^{I} \setminus W_{zgr}^{II}\right) \cup W_{zgr}^{III} \tag{23}$$

where:

W_{zgr}^{I} - the sum of the sets of nodes of the "road" type;

W_{zgr}^{II} - set of the crossing nodes (see sect. 1.3 and tab. 1, see also [10], [11]);

W_{zgr}^{III} - subset of W_{zgr}^{II}, which represents only one of the nodes for each "crossing" type node;

Set U_{zgr} of arcs is created as follows :

$$\underset{j=1,L_{lin}^{1}}{\forall} \; \underset{\substack{a,b=1,\left|W_{lin}^{1,j}\right| \\ a \neq b}}{\forall} \; (u_{a,b} \in U_{lin}^{1,j}) \Rightarrow (u_{a,b} \in U_{zgr} \wedge u_{b,a} \in U_{zgr}) \tag{24}$$

where :

$$u_{a,b} = u \in U_{lin}^{1,j} : \langle w_a, u, w_b \rangle \in P_{lin}^{1,j} \tag{25}$$

In short, if for each graph which describes "road" type object exists an edge incident to the nodes w_a and w_b, then in G_{zgr} this edge is replaced by two arcs with the opposite directions.

Function described on the graph nodes is following:

$$\underset{j=1,L^1_{lin}}{\forall} \quad \psi^1_{zgr}(w^j_g) = FW^{1,j}_{lin}(w^j_g) \Leftrightarrow w^j_g \in W^{1,j}_{lin} \tag{25}$$

Set of functions described on the graph's arcs looks as follows :

$$\zeta_{zgr} = \left\{\zeta^1_{zgr}(\cdot), \zeta^2_{zgr}(\cdot), \ldots, \zeta^{J_{zgr}}_{zgr}(\cdot)\right\} \tag{26}$$

$$\underset{j=1,L^1_{lin}}{\forall} \quad \zeta^1_{zgr}(u_{a,b}) = \xi^{1,j}_2(u_{a,b}) \Leftrightarrow u_{a,b} \in U^{1,j}_{lin} \tag{27}$$

where:

ζ^1_{zgr} - describes topographical characteristics of the road (as in the mathematical terrain model, see (19)) ;

ζ^2_{zgr} - function describing a condition traffic coefficient which effects speed of movement, depending on : surface of the road, road width, kind of terrain, weather, year season, etc., $\zeta^2_{zgr} : U_{zgr} \rightarrow (0, 1]$;

ζ^j_{zgr} - other functions described on the graph's arcs, $j = \overline{3, J_{zgr}}$;

Description of functions above presented results from, for example tactical parameters of terrain, and may be achieved from various military rules and norms ([5], [7], [8]) or may be described by experts [9].

2.2 Model of the precise TRN

Precise TRN model differs in many detail from the rough TRN model. In this model we can't use "ready" vertices (as in the rough TRN, where the vertices were given from the graphs describing the "road" type objects) because we want to fix trafficable routes accross terrain (including possibility of traffic on the existing roads). A discrete terrain description has been proposed. The idea is, the partition of terrain area by means of square grid which create so-called quadtrees [5].

Partition of topographical area gives some advantages:

a) it enables to take into consideration topographical elements of the terrain essential from the point of view of possibility terrain traffic (by decreasing only topographical heterogeneous areas to gain some contractual topographical homogeneous ones);
b) in the case of a regular square grid we might use a thick grid in order to obtain the same effect as in a);
c) various size elements afford shorten data notation.

We part some terrain square area into four smaller squares until the obtained square subareas will contain a homogeneous topographical information (on the fixed detail level). These squares we call the squares of topographical homogeneous area (STHA).

<u>*Definition 1*</u>

The square of topographical homogeneous area (STHA) is such a terrain area (which create a square – element of the quadtree) which have a contractual topographical homogeneous properties (for terrain relief and its covering) considering possibility of military units' movement in this area.

Such a homogeneous area is: forest, lake, and other areas which have a homogeneous terrain relief or (and) its covering.
We define the square of the considered area as follows :

$$K_\chi = w_\chi \in WSP \tag{28}$$

where :

K_χ - χ-th square of area;

w_χ - coordinates of the left lower corner of χ square;

We can evaluate length a_χ of the square side as follows :

$$a_\chi = \frac{a}{2^{l_\chi}} \tag{29}$$

where:

a - side length of the primary square (of the root);

l_χ - partition depth of the primary square (number of digits of χ-th square denoted in quadtree system [3]);

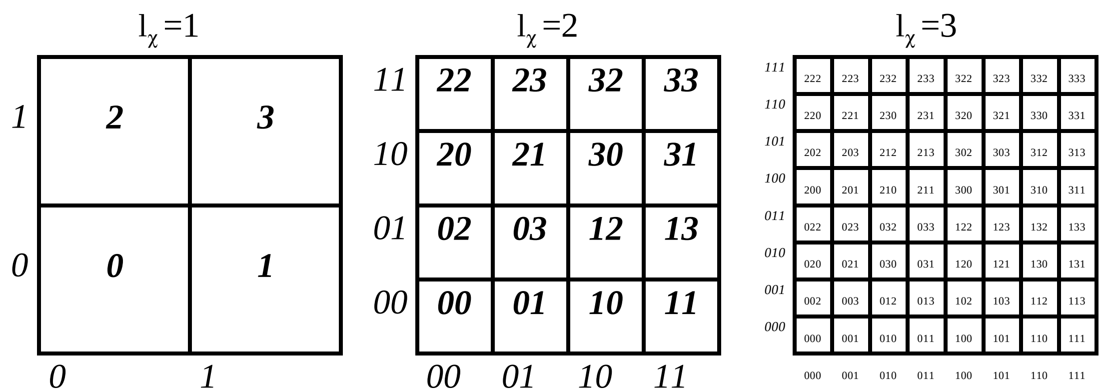

Fig. 1 Description of squares in quadtree for l_χ=1, 2, 3

Before we present algorithms for precise TRN constructing, we must notice some fact. In reality the terrain may be presented as three elements :

- relief ;
- grounds ;
- covering ;

The primary terrain element is the relief. Next are grounds and the last – is covering. This fact suggests how to construct the algorithms.
First, we will part the terrain considering its relief (algorithm 2.1). After this we obtain some numbers of relief homogeneous square subareas of the considered terrain area. Considering the fact that relief isn’t a terrain element which homogeneity can be easily fixed (as e.g. for forest, lake or other covering elements), it has been assumed that as the relief homogeneous area we understand such one where the biggest sloping angle between two points of this area is in some limits.
When we have partitioned the terrain on the relief homogeneous squares (areas) then we fix homogeneity of these ones considering the grounds and the covering for each of these squares (algorithm 2.2) [10], [11].
Denotations used in the algorithms :

$Z_{RPojazd}$ - set of attribute vectors of military unit types;

R_{pojazd} - number of military unit types ;

Z_{obiekt} - set of areas, linear and point objects, which are (downright or in part) in the area of the considered square;

a - side length of the primary square;

s_p - length of the p-th type unit, $p = \overline{1, R_{pojazd}}$;

l_p - „margin” beetwen the square side and the p-th type unit;

a^{min} - minimum square side length, $a^{min} = s_p + 2 \cdot l_p$;

$Z_{KNachyl}$ - set of vectors describing terrain sloping angles;

$Z_{sprawdz}$ - set of numbers of squares (denoted in quadtree system), to be checked;

$Z_{KwRzeźba}$ - set of numbers of squares created after partitioning of the terrain area on the relief homogeneous squares;

χP - number of primary squares (which we start the partition from);

Z_{RNach} - set of sloping angle types for the fixed χ square, evaluated based on the sloping angles between pair of points which are inside χ square; $Z_{KNachyl}$ set ($|Z_{RNach}| > 1$ describes that there is more than one type of terrain sloping angles inside χ square) ;

$X\chi$ - set of numbers of all squares describing considered terrain;

a_χ - side length of square χ ;

$K_\chi = \left\langle x_\chi, y_\chi, z_\chi \right\rangle$ - coordinates of the left lower corner of square χ ;

$K_\chi^{śr} = \left\langle x_\chi + \frac{1}{2} a_\chi, y + \frac{1}{2} a_\chi, z_\chi^{śr} \right\rangle$ - coordinates of the centre of square χ ;

$$F_{ij}(\chi_i, \chi_j) = \begin{cases} 1 \ , & \text{if the terrain belt with the width } (s_p + l_p) \\ & \text{is trafficabled from the centre of } \chi_i \text{ to the centre of } \chi_j \\ 0 \ , & \text{otherwise} \end{cases}$$

$Z\chi_i$ - set of number of squares to which a direct route exists from square χ_i ;

$X'_{\chi i}$ - ordered set of numbers of squares neighbouring with square χ_i ;

Algorithm 2.1 (Partitioning of terrain area on relief homogeneous area squares for the p-th kind of military unit)

1°. $Z_{sprawdz} := \{\chi P\}$, $Z_{KwRzeźba} := \varnothing$, $Z_{RNach} := \varnothing$.

2°. $Z_{sprawdz} = \varnothing$, yes or not ?

If YES then jump to **6°.**

If NOT (that is $Z_{sprawdz} \neq \varnothing$) then take an element from $Z_{sprawdz}$ and denote it by χ.

Is side length a_χ of square χ less or equal to the minimum side length a^{min} for the p-th type unit?, that is

Is $a_\chi \leq a^{min}$?

If YES then go to **4°**, otherwise to **3°**.

3°. Create Z_{RNach} set.

Is $|Z_{RNach}| > 1$?

If YES (that is χ square isn't homogeneous considering terrain relief) then go to **5°**.

If NOT then go to **4°**.

4°. $Z_{KwRzeźba} := Z_{KwRzeźba} \cup \{\chi\}$, $Z_{sprawdz} := Z_{sprawdz} \setminus \{\chi\}$ and go to **2°**.

5°. Part the current χ square on four less squares (see Fig.1).

Increase $Z_{sprawdz}$ set of not checked squares as follows :

$$Z_{sprawdz} := (Z_{sprawdz} \cup \{\chi0, \chi1, \chi2, \chi3\})$$

where :

$\chi0, \chi1, \chi2, \chi3$- numbers of new squares created by adding, at the tail of χ square, of digits 0, 1, 2 i 3 (see Fig.1).

NOTICE !

If χ was the primary square then $\chi0 = 0$, $\chi1 = 1$, $\chi2 = 2$, $\chi3 = 3$.

Next, set the coordinates of the left bottom corners of new squares :

$$K_{\chi0} = \langle x_\chi, y_\chi, z_{\chi0} \rangle \qquad K_{\chi2} = \langle x_\chi, y_\chi + \tfrac{1}{2}a_\chi, z_{\chi2} \rangle$$

$$K_{\chi1} = \langle x_\chi + \tfrac{1}{2}a_\chi, y_\chi, z_{\chi1} \rangle \qquad K_{\chi3} = \langle x_\chi + \tfrac{1}{2}a_\chi, y_\chi + \tfrac{1}{2}a_\chi, z_{\chi3} \rangle$$

Delete square χ from $Z_{sprawdz}$:

$$Z_{sprawdz} := Z_{sprawdz} \setminus \{\chi\}$$

$$Z_{RNach} := \varnothing$$

Go to **2°**.

6°. END.

<u>Algorithm 2.2</u> (Partitioning of terrain area on grounds and covering homogeneous area squares for: the p-th kind of military unit, given weather conditions and year season)

1°. $Z_{obiekt} := \varnothing$;

Is $Z_{KwRzeźba} = \varnothing$?

If YES then go to **7°**.

If NOT, then take an element of $Z_{KwRzeźba}$ and denote it by χ.

$Z_{sprawdz} := \{\chi\}$.

2°. The same as point **2°** in algorithm 2.1.

3°. Designate the Z_{obiekt} set as follows :

$$\underset{i=1,R_{pow}}{\forall} \ \underset{j=1,L^i_{pow}}{\forall} \ (O^{i,j}_{pow} \ in \ \chi \Rightarrow Z_{obiekt} := Z_{obiekt} \cup \{O^{i,j}_{pow}\})$$

Notation „$O^{i,j}_{pow}$ *in* χ” reads: there is (downright or in part) the j-th area object of the i-th type inside χ square.

Is $|Z_{obiekt}| \geq 2$? (i.e., are there at least two various area objects inside χ square)

If YES then go to **5°.**

Otherwise increase the set Z_{obiekt} :

$$\underset{i=1,R_{pkt}}{\forall} \ \underset{j=1,L^i_{pkt}}{\forall} \ (O^{i,j}_{pkt} \ in \ \chi \Rightarrow Z_{obiekt} := Z_{obiekt} \cup \{O^{i,j}_{pkt}\})$$

Is $|Z_{obiekt}| \geq 2$?

If YES then go to **5°.**

Otherwise increase the set Z_{obiekt} :

$$\underset{i=1,R_{lin}}{\forall} \ \underset{j=1,L^i_{lin}}{\forall} \ (O^{i,j}_{lin} \ in \ \chi \Rightarrow Z_{obiekt} := Z_{obiekt} \cup \{O^{i,j}_{lin}\})$$

Is $|Z_{obiekt}| \geq 2$?

If YES then go to **5°.**

Otherwise (χ square is topographicaly homogeneous) go to **4°**.

4°. Execute algorithm 2.3. After this do :

$Z_{sprawdz} := Z_{sprawdz} \setminus \{\chi\}$

Go to **2°**.

5°. The same as point **5°** in algorithm 2.1 with one change : instead $Z_{RNach} := \emptyset$ we set

$Z_{obiekt} : = \emptyset$.

6°. $Z_{KwRzeźba} := Z_{KwRzeźba} \setminus \{\chi\}$

Go to **1°**.

7°. END.

Algorithm 2.3 (Determination of traffic possibility by topographical homogeneous square or by square with the minimum side length)

1°. Is $a_\chi > a^{min}$?

If YES then jump to **2°**.

If NOT then jump to **3°**.

2°. Set the value of a function which describes velocity weakening coefficient for the p-th type unit inside square χ :

$$F^p_{oslab}(ob) := f^{\,p}_{atm}(ob, atm) \cdot f^{\,p}_{topogr}(ob)$$

where :

$F^p_{oslab} : Z_{obiekt} \rightarrow [0,1]$;

Z_{atm} - set of vectors which coordinates describe weather condition;

$f^{\,p}_{atm}(ob, atm)$ - function describing effect of weather condition and year season "atm" on decreasing traffic possibility by χ square described by object attributes $ob \in Z_{obiekt}$, for the p-th unit type, $f^{\,p}_{atm}(ob, atm) : Z_{obiekt} \times Z_{atm} \rightarrow [0,1]$;

$f^{\,p}_{topogr}(ob)$ - function describing velocity weakening for the p-th unit type inside χ square which is described by object attributes $ob \in Z_{obiekt}$. Value of this function results from topographical properties of terrain only, $f^p_{topogr}(ob) : Z_{obiekt} \rightarrow [0,1]$;

Go to **5°**.

3°. Check the condition :

$$ob \in Z_{obiekt} \,\&\, \left(\underset{j=1,L^1_{lin}}{\exists} ob = O^{1,j}_{lin} \text{ or } \underset{j=1,L^3_{lin}}{\exists} ob = O^{3,j}_{lin} \right)$$

If the above condition is satisfied then do

$$(\, ś_{ob} \ \textit{in} \ \ \chi \,) \Rightarrow (\, F^p_{oslab}(ob) > 0 \,)$$

where $ś_{ob}$ – centre of linear object. Go to **5°**.

Otherwise, go to **4°**.

4°. Check, whether at least one trafficable route exists with the width equals to or greater than a^{min}, from the centre of square to the centre of some side of corner of this square.

If YES then

$$F^p_{oslab}(ob) := f^{\,p}_{atm}(ob, atm) \cdot f^{\,p}_{topogr}(ob),$$

If NOT then

$$F^p_{oslab}(ob) := 0 \,;$$

Go to **5°**.

5°. END.

Algorithm 2.4 (Determination of possible links between squares)

0°. $i := 1;$
$Z\chi_i := \varnothing$;

1°. Take the i-th ($i = \overline{1,|X\chi|}$) square χ_i from the set $X\chi$ and check :

Is $F^{p}_{oslab}(ob) > 0$?

If YES, create set $X'_{\chi i}$. Add, to the set of numbers of squares, from which exists a direct route to square χ_i , such of squares as follows :

$$\underset{\substack{j \in X'\chi_i \\ j \neq i}}{\forall} (F_{ij}(\chi_i, \chi_j) = 1) \Rightarrow \left(Z\chi_i := Z\chi_i \cup \{\chi_j\}\right)$$ and go to **2°**.

If NOT (namely, square χ_i is not trafficable) then
$i := i+1;$
$Z\chi_i := \varnothing;$

Go to **1°**.

2°. END.

Example of application of algorithms 2.1÷2.4 for the partition of topographical map information on STHA is in Fig.2. In Fig.3 the real terrain is presented and its partition on STHA. Possible links between neighbouring squares for terrain area in Fig. 3 are in Fig.4.

After partitioning of terrain on STHA (algorithms 2.1, 2.2 and 2.3) and after determination of possible links between neighbouring squares (algorithm 2.4) the precise trafficable routes network may be presented as follows:

$$S_{dok} = \left\langle G_{dok}, \psi^{i}_{dok}, \zeta^{j}_{dok} \right\rangle \tag{30}$$

where:

G_{dok} - Berge graph describing trafficable routes network;

ψ^{i}_{dok} - set of functions described on the graph vertices, $i = \overline{1, I}_{dok}$;

ζ^{j}_{dok} - set of functions described on the graph arcs, $j = \overline{1, J}_{dok}$;

We define :

$$G_{dok} = \left\langle W_{dok}, U_{dok}, P_{dok} \right\rangle \tag{31}$$

where:

W_{dok} - set of graph vertices;

U_{dok} - set of graph arcs;

P_{dok} - relation, $P_{dok} \subset W_{dok} \times U_{dok} \times W_{dok}$;

Elements of G_{dok} are:

$W_{dok} = X\chi$ - set of graph vertices equals the set of all squares of partitioning of the terrain area;

Set U_{dok} is :

$$\underset{i=\overline{1,X\chi}}{\forall} \; \underset{\substack{j=\overline{1,X\chi} \\ j \neq i}}{\forall} (u_{i,j} \in U_{dok}) \Leftrightarrow j \in Z\chi_i \tag{32}$$

Fig. 2 Partition of the selected map area on squares of topographical homogeneous areas (STHA)

Denotation (32) says that arc $u_{i,j}$ which links vertex (square) χ_i with χ_j belongs to U_{dok} when possibility of traffic from χ_i to χ_j exists (i.e. $j \in Z\chi_i$).

Functions described on the set of vertices are defined in the following manner:

$$\psi^{i}_{dok} = \left\{\psi^{1}_{dok}\right\} \quad (33)$$

$$\psi^{1}_{dok} : W_{dok} \rightarrow K^{śr} \quad (34)$$

where:

$$K^{śr} = \left\{K^{śr}_{1}, K^{śr}_{2}, \ldots, K^{śr}_{\chi}, \ldots, K^{śr}_{|X\chi|}\right\} \quad (35)$$

Set of functions described on graph arcs is:

$$\zeta^{j}_{dok} = \left\{\zeta^{1}_{dok}, \zeta^{2}_{dok}, \ldots, \zeta^{J_{dok}}_{dok}\right\} \quad (36)$$

$$\zeta^{1}_{dok} : U_{dok} \rightarrow (0,1] \quad (37)$$

$$\zeta^{1}_{dok}(u_{i,j}) = \frac{a_{\chi_i} \cdot F^{p}_{oslab}(ob_{\chi_i}) + a_{\chi_j} \cdot F^{p}_{oslab}(ob_{\chi_j})}{a_{\chi_i} + a_{\chi_j}} \quad (38)$$

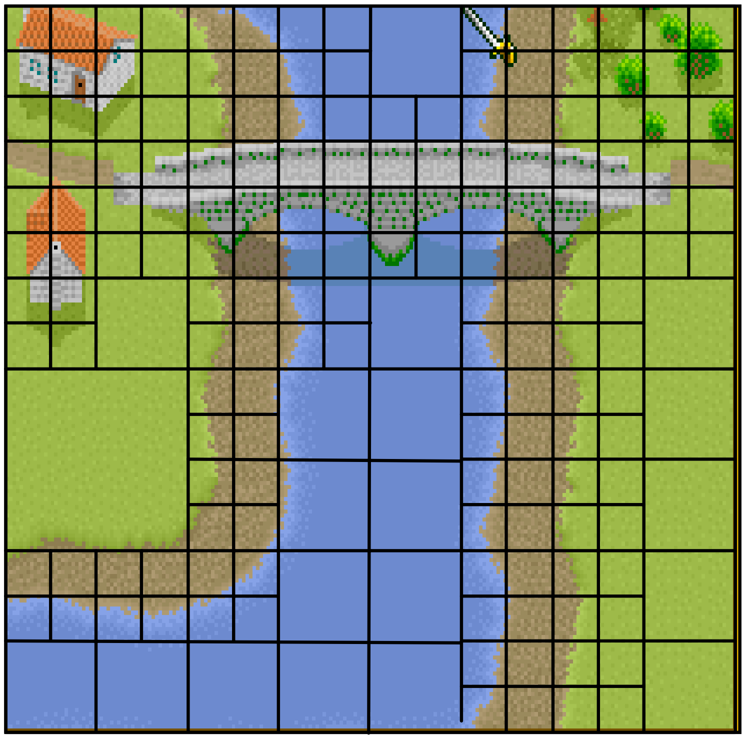

Fig. 3 Partition of the selected real terrain area on squares of topographical homogeneous areas (STHA)

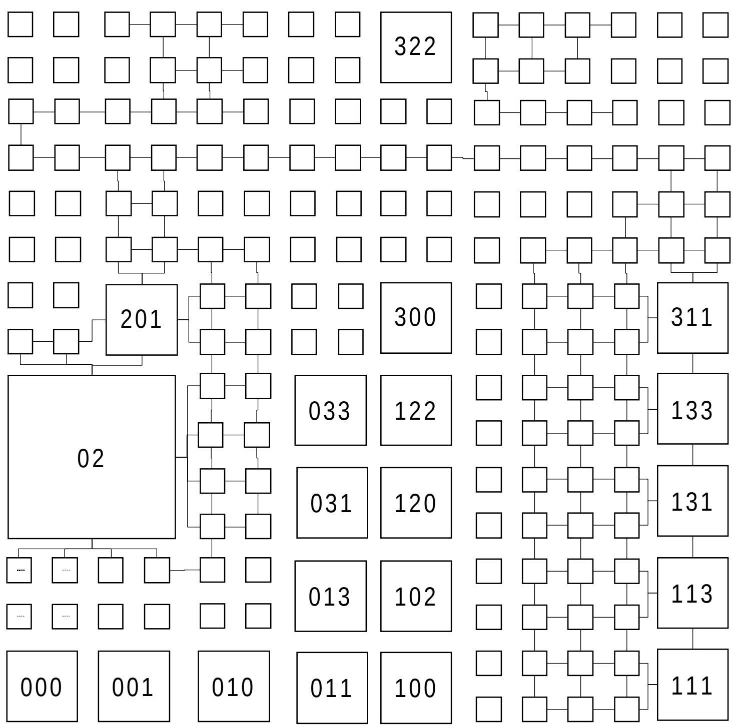

Fig. 4 Determination of possible links between neighbouring squares and description of selected vertices in quadtree system for terrain area presented in Fig. 3

Function ζ_{dok}^{1} for arc $u_{i,j}$ linking square χ_i with χ_j, is equal to the weighted average of function value $F_{oslab}^{p}(ob_{\chi_i})$ describing square (vertex) χ_i and $F_{oslab}^{p}(ob_{\chi_j})$ describing square (vertex) χ_j (see algorithm 2.3).

$\zeta_{dok}^{2}, \zeta_{dok}^{3}, \ldots$ - functions, which may describe, for example level of opponents activities on considered graph arcs, efficiency of these activities,etc.

Summary

We notice that the terrain partition is carried out with various detail levels (using algorithms 2.1÷2.4). This level principally depends on: the size of contractual least square of topographical homogeneous area and contractual unit width for which we create a precise TRN. The more terrain is differentiated the more complex is the precise network. Simultaneously, application of unregular square size allows for decreasing of computer memory size. Additionally, we may try to merge squares of topographical homogeneous area into bigger squares (decreasing the detail level) basing on, for example [1], [2].

Taking into consideration the approach concerning a rough TRN, it makes into possible reduction of the network (when the precise network is not necessary; in this way we simultaneosuly reduce the size of all problems solved on this network, e.g. the problem of finding a shortest path, etc.) or brings into relief a many of terrain details. Moreover, depending on detail level, we can consider only the roads of: first categories, first and second categories, first and second and third, etc. The complexity of actual problem effects the kind of the network model which we must choose.

The presented network models have been utilized to movement simulation [11] and redeployment planning [12]. They may also be utilized for any problems concerning transport planning, and the mathematical terrain model may be the base to build terrain database structure.

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Modelowanie terenu dla potrzeb symulacji przemieszczania środków walki

Rozpatrzono problem pozyskiwania informacji terenowej wykorzystywanej w algorytmach symulacji przemieszczania obiektów. Przedstawiono matematyczny model terenu bazujący na informacji z mapy topograficznej. Model ten wykorzystano na etapie konstrukcji sieci tras przejazdu (STP). Zdefiniowano dwa rodzaje sieci tras przejazdu : zgrubną i dokładną. Pierwszy typ sieci oparto o rzeczywiste drogi istniejące w terenie. Sieć dokładna bazuje na podziale terenu na kwadraty obszarów jednorodnych topograficznie. Idea tego podziału została oparta o model drzewa czwórkowego. Zaprezentowano przykłady wykorzystania przedstawionych modeli do tworzenia sieci tras przejazdu.