THE METHOD OF EVALUATION THE CHARACTERISTICS OF CONFLICT SITUATION MODELLED BY OPERATIONAL GAME

ZBIGNIEW TARAPATA

Institute of Mathematics and Operations Research
Faculty of Cybernetics
Military University of Technology
Kaliskiego Street 2 P-01-489 Warsaw
Tel. (022)-685-75-66
email : ztarap@isi.wat.waw.pl

A simplified mathematical model of hipothetic conflict situation is presented. As conflict situation are considered a conflict between two sides which have a some resources of military objects. This conflict is modelled on operational game. Two types of conflict characteristics, output and input-output, are defined. We propose the method of statistical analysis of simulation output data to estimate interesting characteristics. The control variable method to reduction estimator variance of probability distribution function parameters of estimation characteristics are used. We showed a some results of statistical data analysis and method for aiding decision making of players.

Introduction

Combat actions aren't easy object for research in reality. For obtain an information about procedure in different conflict situations without real suffer losses we propose a construction of simulation model of conflict situations and next research a different of conflict situation variants in this model. The model of situation which we described is operational game (simulation game).

The idea of the problem is following:

Two oppose sides, **A** and **B**, are specified. Each of their have a military object resources. In consequence decision (move) of player representing the side of conflict can suffer losses or profits. Each of decision of first player cause to reply of second player. We establish that each of decision is making without acquaintance of opponent decision as a result of this situation is making a random decision and players receive information about the results. The game can be finally by reason of combat fatigue resources for one of the side or by obtain appointed level or profits (losses).

Presented a conflict situation is a clean hipothetic.

The first goal of this model is not only evaluation of characteristics this concrete situation but presenting the general idea of evaluation the characteristics. The second goal is aiding decision making for players.

In [8] are considered two advances : first - **A** player is a human which play with computer (**B** player) and he don't know realization of decision of **B** player, second - gaming with full information for both players, that is they can observe their moves.

This paper is organized as follows.

In section **1** a simplified mathematical model of conflict situation is presented. In section **2** the characteristics of conflict situation are defined. Section **3** contains the method of evaluation these characteristics. A method of researching conflict situation is presented in section **4**. In the last section we showed a some results and a method of aiding decision making of players, basing on simulation experiments.

1. A mathematical model of conflict situation

We establish, that resources for both conflict sides are different types of military objects and its are grouped in detachments. Each of detachments contains the same type of military objects. We acceptance that „combat strength" of detachments is evaluation as follows:

$$\Phi_A^{s,g}(t) = N_A^{s,g}(t) \cdot E_A^i \cdot K_A^{s,g}(t) \cdot W_A^{s,g}(t)\,, \tag{1}$$

$$\Phi_B^{q,f}(t) = N_B^{q,f}(t) \cdot E_B^j \cdot K_B^{q,f}(t) \cdot W_B^{q,f}(t)\,, \tag{2}$$

where :

- $\Phi_A^{s,g}(t)$ - combat strength of s-th detachment of **A** player in battle with g-th detachment of **B** player in the moment t;
- $\Phi_B^{q,f}(t)$ - combat strength of q-th detachment of **B** player in battle with f-th detachment of **A** player in the moment t;
- $N_A^{s,g}(t)$ - number of military objects in s-th detachment of **A** player, which fight with g-th detachment of **B** player in the moment t;
- $N_B^{q,f}(t)$ - number of military objects in q-th detachment of **B** player, which fight with f-th detachment of **A** player in the moment t;
- $K_A^{s,g}(t)$ - ratio which describes type of combat action effect on combat strength for s-th detachment of **A** player which fight with g-th detachment of **B** player in the moment t ;
- $K_B^{q,f}(t)$ - ratio which describes type of combat action effect on combat strength for q-th detachment of **B** player which fight with f-th detachment of **A** player in the moment t ;
- $W_A^{s,g}(t)$ - ratio which describes s-th detachment of **A** player from g-th detachment of **B** player distance effect on combat strength of s-th detachment of **A** player in the moment t;
- $W_B^{q,f}(t)$ - ratio which describes q-th detachment of **B** player from f-th detachment of **A** player distance effect on combat strength of q-th detachment of **B** player in the moment t;
- E^i_A, E^j_B - ratio of effectivity for i-th (j-th) types of military object of **A**(**B**) player in relation to effectivity of unrealistic „model" military object , i =1..3 (j =1..3);

A moment, for example t_p, is the moment of appearance one of events in the system, which may be e.g. making the next shot by one of the sides. In this case in the interval time t_p-t_{p-1} the system will not vary. If $\Phi_A^{s,g}(t_{p-1}) > \Phi_B^{q,f}(t_{p-1})$ then **A** player will win otherwise **B** player will win. If $\Phi_A^{s,g}(t_{p-1}) = \Phi_B^{q,f}(t_{p-1})$ then players lose the same number of military objects. The number of saved military objects in t_p for detachments of each players we following evaluate :

$$N_A^{s,g}(t_p) = N_A^{s,g}(t_{p-1}) - \left(N_A^{s,g}(t_{p-1}) \cdot 2^{\frac{-\Phi_A^{s,g}(t_{p-1})}{\Phi_B^{q,f}(t_{p-1})}} \cdot \frac{t_p - t_{p-1}}{t_p} \right),\ t_0 = 0,\ p \geq 1 \tag{3}$$

$$N_B^{q,f}(t_p) = N_B^{q,f}(t_{p-1}) - \left(N_B^{q,f}(t_{p-1}) \cdot 2^{\frac{-\Phi_B^{q,f}(t_{p-1})}{\Phi_A^{s,g}(t_{p-1})}} \cdot \frac{t_p - t_{p-1}}{t_p} \right),\ t_0 = 0\ ,\ p \geq 1 \tag{4}$$

i.e. the percently losses $U_A^{s,g}(\Delta t)$ of military objects for s-th detachment of **A** player which fight with g-th detachment of **B** player in time interval $\Delta t = t_p - t_{p-1}$, is equal :

$$U_A^{s,g}(\Delta t) = \frac{N_A^{s,g}(t_{p-1}) - N_A^{s,g}(t_p)}{N_A^{s,g}(t_{p-1})} \cdot 100\% \tag{5}$$

Additionally for „monitoring" state of the game we propose an estimate balance of profits and losses:

$$BS_A(t) = \sum_{s=1}^{\mathbf{a}} SS_A^{R_A^s} \cdot (N_A^s(0) - N_A^s(t)), \tag{6}$$

$$BK_A(t) = \sum_{q=1}^{\mathbf{b}} SK_A^{R_B^q} \cdot (N_B^q(0) - N_B^q(t)), \tag{7}$$

and

$$N_A^s(t) = N_A^{s,g}(t) \cdot l_A^s(t) \tag{8}$$

$$N_B^q(t) = N_B^{q,f}(t) \cdot l_B^q(t) \tag{9}$$

where:

$BS_A(t), BK_A(t)$ - balance of losses and profits respectively of **A** player to moment t (for **B** player by analogy);

$N_A^s(t)$ - total number of military objects in s-th detachment of **A** player in the moment t, ($N_A^s(0)$ describes a total number of military objects in s-th detachment of **A** player on start of the game);

$N_B^q(t)$ - total number of military objects in q-th detachment of **B** player in the moment t, ($N_B^q(0)$ describes a total number of military objects in q-th detachment of **B** player on start of the game);

SS_A^i - losses ratio for lose of i-th type of military object of **A** player, i =1...$\mathbf{r_A}$;

SK_A^j - profits ratio for **A** player, for damage of j-th type of military object of **B** player, j =1...$\mathbf{r_B}$;

$\mathbf{r_A}, \mathbf{r_B}$ - number of types of military objects for **A** and **B** player respectively ;

$l_A^s(t)$ - number of detachments of **B** player, wherewith fight of s-th detachment of **A** player in t;

$l_B^q(t)$ - number of detachments of **A** player, wherewith fight of q-th detachment of **B** player in t;

a, **b** - total numbers of detachments of **A** and **B** player respectively, $\mathbf{a} = \sum_{i=1}^{\mathbf{r_A}} k_A^i$, $\mathbf{b} = \sum_{j=1}^{\mathbf{r_B}} k_B^j$;

k_A^i - number of i-th type detachments of **A** player, i =1...$\mathbf{r_A}$;

k_B^j - number of j-th type detachments of **B** player, j =1...$\mathbf{r_B}$;

R_A^s - type of military objects wherewith consist of s-th detachment of **A** player, s =1... k_A^i ;

R_B^q - type of military objects wherewith consist of q-th detachment of **B** player, q =1... k_B^j ;

2. A characteristics of conflict

A two class of characteristics are estimated :

⇒ output ;

⇒ input-output ;

To first class we among the following characteristics :

a1) lose of player resources for fixed moment t , $X^U(t)$;

b1) time of player lose in the game , X^C ;

c1) general balance on final of the game = profits - losses, X^B (see (6) and (7)) ;

Second class of the characteristics are following :

a2) interdependence between initial state of player resources and final result (general balance) of the game ;

b2) interdependence between strategy of player and final result of the game (assignation which of player strategies is better for obtaining more profitable of general balance on final of the game);

c2) a interval of the time between intelligence reports effect on final result (general balance) of the game;

Applying simulation experiments we will obtain sequence values of research characteristics. This sequence is a some random sample which selected from general population of all results of experiment for observed characteristics. Assurancing an appropriate conditions for experiment we can estimate the characteristics by evaluation of value for selecting point and interval estimators.

3. Planing of experiment and the method of estimating the conflict characteristics

One of more important elements of simulation experiment planing is estimating of number of experiment repetitions, which have effect on size of random sample. All characteristics are function of random variables and their quality depend on size of analysed sample. Number of experiment repetitions so have effect on quality of analysis experiment results.

Identification of the distribution type for examined characteristics is also important.

3.1 Identification of the characteristics distribution type

We establish, that we obtained realization of the random sample for value of selected characteristic X (X^U, X^B or X^C). This is a accomplished sequence of numbers :

$$x_1, x_2, \ldots, x_i, \ldots, x_n. \tag{10}$$

We transform this sequence to sequence with increasing order, that is

$$x^*_1, x^*_2, \ldots, x^*_i, \ldots, x^*_n, \tag{11}$$

for which

$$x^*_{i+1} \geq x^*_i, \qquad i = \overline{1, n-1} \tag{12}$$

Next we partition an observations $x^*_1, x^*_2, \ldots, x^*_i, \ldots, x^*_n$ on R disjoined classes, $R < n$, making a distributive series, wherewith we can obtain an empirical distribution $F_n(x)$:

$$F_n(x) = \begin{cases} 0 & \text{for} \quad x \leq x_1^s \\ \dfrac{m_r}{n} & \text{for} \quad x_r^s < x \leq x_{r+1}^s \\ 1 & \text{for} \quad x_R^s < x \end{cases} \tag{13}$$

where :

x_r^s - middle of r-th class ;

m_r - number of elements for r-th class, with those $\sum_{r=1}^{R} m_r = n$.

We describe by F_0 theoretical distribution of general population. We will check a hypothesis H: $F_0 = F_n$. We can show, that following statistic ([2],[3]):

$$\sum_{r=1}^{R} \frac{\left(m_r - n\cdot\left(F_n(x^s_{r+1}) - F_n(x^s_r)\right)\right)^2}{n\cdot\left(F_n(x^s_{r+1}) - F_n(x^s_r)\right)} \tag{14}$$

for a large n and when we establish that hypothesis H is true proceed to χ^2-Pearson distribution with R-1 degrees of freedom. Limitation for this method are following: $n \geq 30$ and $m_r \geq 5$, $r = \overline{1,R}$. A critical area for Pearson test is $R_\alpha = (\chi^2_{1-\alpha}, \infty)$, where $\chi^2_{1-\alpha}$ is a quantile of χ^2 distribution with R-1 degrees of freedom and confidential ratio α.

A following types of theoretical distribution to approximation of empirical distribution are used : normal, gamma, beta, exponential, uniform, t-Student, Chi-2, powered, Weibull, Snedeckor.

3.2 An estimate of characteristics distribution parameters

To identified a type of analysed sample distribution we must estimate a parameters of this distribution. We used a likelihood ratio method because it give the best of estimators (that is estimators which have the best of properties) [1], [3], [4]. Let general population distribution (e.g. general balance of player on final of the game) be a normal distribution with a probability density function :

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty \tag{15}$$

We show how, basing on n-elements sample, we can estimate by a likelihood ratio method a general population distribution parameters μ and σ.
We create a likelihood function :

$$L(x_1,\ldots,x_n;\mu,\sigma) = \underbrace{\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \cdot\ldots\cdot \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}}_{n} \tag{16}$$

so

$$L(x_1,\ldots,x_n;\mu,\sigma) = \frac{1}{\sigma^n(\sqrt{2\pi})^n} e^{\frac{-1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2} \tag{17}$$

This function describes a probability or density function to obtain a results ($x_1,\ldots,x_n$), if parameters from which a distribution depend are equal $Q_1,\ldots,Q_k$ (in this case: $Q_1=\mu$, $Q_2=\sigma$).

An estimators of parameters Q_i are these values of $\hat{Q}_i$, that likelihood function (17) have a maximum.

If function (17) is differentially with regard to Q_i, $i = \overline{1,k}$, then evaluating estimators of parameters Q_i can be done using a differential calculus. If we follow up the fact that a function

$$\mathbf{ln}\, L(x_1,\ldots,x_n;Q_1,\ldots,Q_k) \tag{18}$$

have a maximum for the same arguments like (17) then we can write :

$$\mathbf{ln}\, L(x_1,\dots,x_n;\mu,\sigma) = -n\,\mathbf{ln}\sigma - \frac{n}{2}\,\mathbf{ln}2\pi - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2 \qquad (19)$$

The wanted estimators we evaluating from following system of equations :

$$\begin{cases} \dfrac{1}{\sigma^2}\displaystyle\sum_{i=1}^{n}(x_i-\mu) = 0 \\ -\dfrac{n}{\sigma} + \dfrac{1}{\sigma^2}\displaystyle\sum_{i=1}^{n}(x_i-\mu)^2 = 0 \end{cases} \qquad (20)$$

Calculating (20) we obtain :

$$\begin{aligned} &\sum_{i=1}^{n} x_i - n\mu = 0 &&\Rightarrow \overset{\wedge}{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i = \overline{X_n}, \\ &n\sigma^2 - \sum_{i=1}^{n}(x_i-\mu)^2 = 0 &&\Rightarrow \overset{\wedge}{\sigma^2} = \frac{1}{n}\sum_{i=1}^{n}(X_i-\overline{X})^2 = S^2 \end{aligned} \qquad (21)$$

that is to say for a normal distribution we obtain, that estimators of parameters μ,σ are mean value and variance from sample, respectively. A likelihood estimators for most often distributions are presented, e.g. in [2, pp. 262÷264].

If we establish, that general population distribution for X characteristic is normal N(μ,σ) (for large repetitions of simulation this is a conclusion from Lindeberg-Levy theorem [1],[2],[4]), then point estimators of mean values for characteristics we can evaluate as follows.

Ad.a1)

Let $X^u_1(t)$, $X^u_2(t)$,...,$X^u_n(t)$ describe losses of resources to t (for fixed t, $X^U(t)$ is a random variable but in generality $X^U(t)$ describe some stochastic process), in successive experiment. If these experiments were n then mean value estimator of losses of resources to the moment t we following evaluate :

$$\overset{\wedge}{\overline{X^U}}(t) = \frac{1}{n}\sum_{i=1}^{n} X_i^u(t)\,, \qquad (22)$$

Ad.b1)

Let X^c_1, X^c_2 ,. . . ,X^c_n describe time of player lose in successive simulation experiments. Because these experiments is n so mean value point estimator of this time we evaluate as follows :

$$\overset{\wedge}{\overline{X^C}} = \frac{1}{n}\sum_{i=1}^{n} X_i^C \qquad (23)$$

Ad. c1)

Let X^B=BK(t_k)–BS(t_k) , where t_k - final time of game, BK(·), BS(·) described by (6) and (7). Let X^B_1,X^B_2 ,. . . ,X^B_n describe a general balance of player in i-th simulation experiment, $i = \overline{1,n}$. Then mean value estimator of a general balance of player is equal :

$$\overset{\wedge}{\overline{X^B}} = \frac{1}{n}\sum_{i=1}^{n} X_i^B \qquad (24)$$

The estimators which have been evaluated above are the point estimators. However we very often should have a some interval for estimator value. We show a method of evaluation a interval estimator of mean value of random variable X (X^U, X^B or X^C) [1],[3].

We describe by $\hat{X}$ mean value estimator of random variable X.

A variance of estimator $\hat{X}$ is equal :

$$S^2\left(\hat{X}\right) = \frac{\sigma^2}{n} \tag{25}$$

where σ^2 is a variance of random variable X, that is $\sigma^2 = D^2(X)$.
A unbiassed estimator of variance is following :

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{X})^2 = \frac{1}{n-1}\sum_{i=1}^{n}X_i^2 - \frac{n}{n-1}\hat{X}^2 \tag{26}$$

Because variance is not known (we evaluate it with sample obtained from simulation) so following random variable *k* :

$$k = \frac{\hat{X} - X}{\hat{\sigma}/\sqrt{n}} \tag{27}$$

have a t-Student distribution with *n*-1 degrees of freedom. We use a variable *k* to construct confidence interval for mean value $\overline{X}$. For fixed confidence coefficient 1-α, from a table of t-Student distribution for *n-1* degrees of freedom we can read of value k_α and next making transformation we will obtain following confidence interval for mean value :

$$P\left\{\hat{X} - k_\alpha \frac{\hat{\sigma}}{\sqrt{n}} < \overline{X} < \hat{X} + k_\alpha \frac{\hat{\sigma}}{\sqrt{n}}\right\} = 1 - \alpha \tag{28}$$

This is important, that when *n* increase then different between t-Student distribution and standard normal distribution is not essential. So for $n \geq 30$ we can replace k_α by u_α from table of normal distribution N(0,1).

Point and interval estimation of variance are presented e.g. in [1], [3], [4], [7], [10].

3.3 Estimation of sample size

Sample size, how we said at the beginning of section **3**, is very important and it effect on accuracy of obtained estimators (22), (23) and (24).
Most often a situation is, that general population distribution is normal N(μ,σ) (for large *n* this is a conclusion from Lindeberg-Levy theorem, how we said earlier), but a both parameters of this distribution not known and we must estimate their from sample. To get to know degree of homogeneous of general population which depend on σ, we must draw initial sample(so-called pilotage sample) which size is n_0 and we must calculate of the following statistic value

$$\hat{\sigma}_z = \sqrt{\left(\frac{1}{n_0 - 1}\sum_{i=1}^{n_0}(X^U{}_i - \overline{\hat{X}^U})^2\right)} \qquad (29)$$

and next, basing on (27) and (28), we finding minimal sample size [1], [2], [10] :

$$n \geq \left(\frac{k_\alpha \hat{\sigma}_z}{g}\right) \qquad (30)$$

where g is a half of confidence interval.

Sample size which has been estimated refers to only one characteristic. If these characteristics is m and for j-th, $j = \overline{1,m}$ we obtain sample size n_j, then total sample size n_c for all characteristics be equal :

$$n_c = \prod_{j=1}^{m} n_j \qquad (31)$$

3.4 A method of reduction an estimator variance

A quality indicator of obtained results is usually an estimator variance and confidence interval of estimated characteristics. Narrowing a confidence interval we can obtain in two ways : enlarging a number of observation or reducing of variance. The first way required elongation of simulation experiment and this is not desirable. Reduction of variance required only appropriate choice of estimator or appropriate organizing process of generating pseudorandom numbers. We propose a control variable method which rely on acquaintance with e.g. mean value of selected observed random variable strongly positive corelated with examined variable through we obtain a reduction of variance. This method we used to checking is possible reduction of variance of finally result (e.g. general balance for player) and as control variable we used a mean value of interval time between sequence of intelligence reports.

Because this system is specific, it was difficultly to find some other estimator for which we known theoretical characteristics, e.g. mean value.

Let, as earlier, $X_1, X_2, X_3, \ldots, X_n$ be sequence of observed random variables (let X describes now time of player lose). We establish, that $X_1, X_2, X_3, \ldots, X_n$ descent from simple random sample. We introduce the notion following [6] :

$\Theta = E(X)$ - real parameter ;

$\sigma_X^2 = Var[X]$ - variance of X random variable;

$\Theta_C = \frac{1}{n}\sum_{i=1}^{n} X_i$ - point estimator ;

$Var(\hat{\Theta}_C) = \frac{\sigma_X^2}{n}$ - estimator variance ;

$S_C^2 = \frac{1}{n(n-1)}\sum_{i=1}^{n}(X_i - \hat{\Theta}_C)^2$ - estimator of variance;

We can define following linear control variable :

$$X_i + C_i = (c_{i1}, c_{i2}, \ldots, c_{iq})^T$$

where:

C_i- control sequence which is a simple random sample, for which a mean value is known and is equal :

$$\mu = (\mu_1, \mu_{2,\ldots,}\mu_q)^T$$

$Z_i = [X_i, C_i^T]^T$, $i = \overline{1,n}$ - answer vector with dimension $(q+1)\times 1$, being a simple random sample;

$$\mathrm{Var}[Z_i] = \begin{bmatrix} \sigma_X^2 & \sigma_{XC} \\ \sigma_{CX} & \sum_{CC} \end{bmatrix}$$

where:

$\sigma_{CX} = \mathrm{cov}_{j=\overline{1,q}}[c_{ij}, X_i]$ - vector with dimension $q\times 1$;

$\sigma_{XC} = \mathrm{cov}_{i=\overline{1,n}}[c_{ij}, X_i]$ - vector with dimension $1\times q$;

$\sum_{CC} = \mathrm{Var}[C_i]$ - matrix (diagonal), when C_i is a simple random sample;

Linear CV-estimator $\hat{\Theta}_L(\Theta = E(Z))$ we following define :

$$\hat{\Theta}_L = \hat{\Theta}_C - (\overline{C} - \mu)^T \hat{\beta} \quad , \quad C = [C_{ij}]_{nxq}, \tag{32}$$

where :

$$\overline{C} = \frac{1}{n}\sum_{i=1}^{n} C_i, \quad \hat{\beta} = S_{CC}^{-1} S_{CX};$$

$$S_{CC} = \frac{1}{n-1}(C^T C - n\overline{C}\overline{C}^T) \ ;$$

where :

$\overline{C}$ - column ; $\overline{C}^T$ - row; $\overline{C}\overline{C}^T$ - square matrix ;

$$S_{CX} = \frac{1}{n-1}(C^T X - n\overline{C}\overline{X}) \ ;$$

$$\overline{X} = (X_1, \ldots, X_n)^T \ ;$$

Estimator of variance $\mathrm{Var}[\hat{\Theta}_L]$ we following evaluate :

$$S_L^2 = S^2\left(\frac{1}{n} + \frac{1}{n-1}(\overline{C} - \mu)^T S_{CC}^{-1}(\overline{C} - \mu)\right), \tag{33}$$

where :

$$S^2 = \frac{1}{n-q-1}\sum_{i=1}^{n}(X_i - \hat{\Theta}_L - (C_i - \mu)^T \hat{\beta})^2 .$$

Confidence interval we describe as follows :

$$\hat{\Theta}_L \pm H_L \quad , \quad H_L = t_{\alpha/2} S_L^2 \quad , \text{ with } n\text{-q-1 degrees of freedom.}$$

We can show, that if some conditions satisfied a variance of new estimator will smaller than variance of examined estimator.

Theorem of Lavenberg-Welch [6]:

We establish, that Z_i is a simple random sample, and Z_i is a vector with dimensions q×1 which have q+1 dimensional normal distribution with vector of mean values $\left(\Theta,\mu^{T}\right)^{T}$ and variance Σ_{ZZ}. We can write

$$E[\hat{\Theta}_L] = \Theta \ , \qquad Var[\hat{\Theta}_L] = \frac{n-2}{n-q-2}(1-R^2)Var[\hat{\Theta}_C]$$

$$E[S_L^2] = Var[\hat{\Theta}_L], \qquad P\{|\hat{\Theta}_L - \Theta| \le H_L\} = 1-\alpha$$

$$R^2 = \frac{\sigma_{XC}\sum_{CC}{}^{-1}\sigma_{CX}}{\sigma_X^2}$$ - square of multiple corellation ratio

Conclusions from theorem L-W

Result from this theorem that estimators $\hat{\Theta}_L$ and S_L^2 are unbiassed and

$$R^2 > \frac{q}{n-2} \Rightarrow Var[\hat{\Theta}_L] < Var[\hat{\Theta}_C]$$

If above inequality is satisfied then we can say that concept of control variable is profitable. Otherwise we must apply $\hat{\Theta}_C$ estimator only.

3.4 A method of evaluation the input-output characteristics

To evaluate the characteristics a2÷c2 which have been described in section **2** we used two-dimensional regression analysis. Nonlinear regression which base on transformation of variable is proposed.

We consider a random vector (ξ,η), which components are appropriate functions of random variables X (e.g. beginning state of player resources) and Y (e.g. general balance of player on final the game) :

$$\xi = h_1(X) \ , \ \eta = h_2(Y) \ .$$

We will want, in order that Eη was linear function of Eξ. After evaluating estimation of regression line

$$\eta = a\xi + b$$

we can apply an invert transformation and we obtain estimation of regression line of random variable Y respect to random variable X in following form

$$y = h_2^{-1}[a h_1(x) + b] \ .$$

Transformation of random variables for some regression functions are showed in ***Tab.1***.

Tab. 1 *Transformation of random variables for examined regression functions*

Form a regression function	**Substitutions**		**Relationships between statistics**	
	ξ	η	*a*	*b*

Form a regression function	Substitutions		Relationships between statistics	
	ξ	η	a	b
$y=cx^{a}$	**lg** X	**lg** Y	a	**lg** c
$y=cr^{x}$	X	**lg** Y	lg r	**lg** c
$y=ax^{n}+b$, n - fixed	X^{n}	Y	a	b
$y=a+b$ **ln** x	**ln** X	Y	a	b
$y=1/(a+be^{-x})$	e^{-x}	Y^{-1}	a	b

Using the least square method we will find estimators of function regression parameters a and b but for X and Y from (34)÷(36) we must take appropriate transformation from ***Tab.1***. And so, for example in case of function $y=cx^{a}$ for X and Y from (34)÷(36) we must substitute **lg** X and **lg** Y, respectively, etc.

$$\hat{a} = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{n}(X_i - \overline{X})^2} = \frac{\sum_{i=1}^{n} X_i Y_i - n\overline{X}\overline{Y}}{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2} \tag{34}$$

$$\hat{b} = \overline{Y} - \hat{a}\overline{X} \tag{35}$$

with those $\overline{X}, \overline{Y}$ are mean values with sequence X and Y respectively. Estimators $\hat{a}$ and $\hat{b}$ are unbiassed and consistent estimators of a and b parameters.
Average deviation on regression line is equal :

$$s = \sqrt{\frac{1}{n-2}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2} \tag{36}$$

with those $\hat{Y}_i$ values are calculating as $\hat{Y}_i = \hat{a}x_i + \hat{b}$.
In finally we must calculate the following statistic

$$t = \frac{\hat{a} - a_0}{s}\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2} \tag{37}$$

which having established a hypothesis H_0: $a=a_0$ have a t-Student distribution with n-2 degrees of freedom. From table of t-Student distribution for fixed confidence level α and for n-2 degrees of freedom we reading t_α , which satisfy $P\{|t|\geq t_\alpha\}=\alpha$. If $|t|<t_\alpha$ is satisfied, then don't need reject the hypothesis H_0.

4. A method of researching conflict situation

In order to research of conflict situation which has been described in **Introduction** the program behavior for simulation game (***SimWar***) has been created which was presented, among other things, on *CEBIT'96* in Hannover. This behavior consist of two part :

1). destined for playing simulation game ;

2). destined for the statistical analysis of the simulation output data respecting for examined characteristics.

In the first part we can set a some parameters respecting to game and both players (number of repetitions; number of human-players; scenario for **B** player; choice or definition strategy of **A** player and other). Strategy for **B** player is included inside of the simulation program so **A** player not know how strategy is used by **B** player. Instead we can set so-called scenario for **B** player, which consist a set of strategies for him random chosen during game by computer dependently on situation in the game. The research were realized with two scenarios of **B** player („active" (ofensive) and „passive" (defensive)) and three strategies of **A** player (defensive, ofensive, combine).

In the second part of this behavior are included options to making statistical data analysis describes in section **3**. For data were assembled in the simulation experiments we can:

- evaluate usual and central moments and median ;
- evaluate a point and interval estimation of variance and mean value ;
- create positive series, distributive series and empirical distribution ;
- check randomness of data sequence from simulation ;
- check aggreament of empirical distribution with one of tens theoretical distributions ;
- estimate, on likelihood ratio method, estimators of distribution parameters ;
- draw diagrams : histogram, empirical distribution, density function and regression function ;
- make a nonlinear regression analysis for estimating second class of characteristics (see sect. **2**).

5. Analysis of the results

We are now present a some results of statistical analysis of the simulation output data. In ***Tab*.*2*** we showed a results of statistical data analysis for general balance of **A** player on final of the game while initial state of resources for both players were identical. We see that different strategies of **A** player were better in relation with different scenarios of **B** player. If **B** player applied his first scenario (defensive) then the third strategy of **A** player was the best for him but for second scenario (offensive) of **B** player the first strategy (defensive) of **A** player turned out the best.
Diagram of histogram of random variable for a general balance of **A** player on final of the game, for his first strategy, first scenario of **B** player and for identical initial state of resources for both players is shown in ***Fig.1***. In ***Fig.2*** we are presented diagram of empirical distribution for random variable presented in ***Fig.1***. We see, that diagram of histogram (***Fig.1***) have a form similar to Gauss curve and it may be a proof that presented random variable have a normal distribution. With analogy a diagram of empirical distribution (***Fig.2***) is similar to diagram of normal distribution.

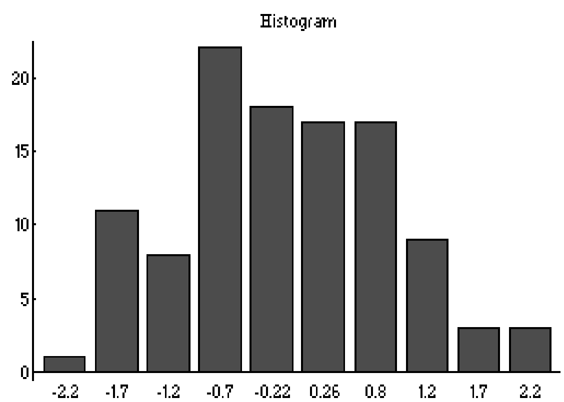
Tab. 2 *The results of realization random variable X^B which describes general balance of* ***A*** *player on final of the game for identical initial states of both players resources. Confidence intervals are calculated for confidence level 0.9. Size of sample n=110.*

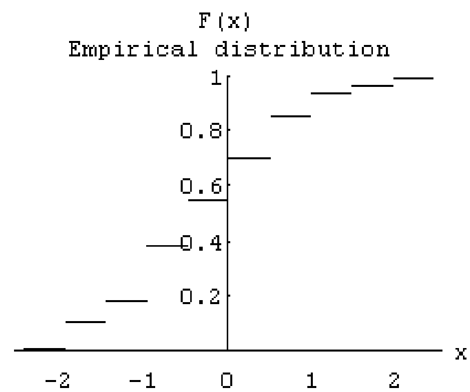
Type of strategy for A player	Type of scenario for B player	Type of distribution	Expectation	Variance	Confidence interval for expectation	Confidence interval for variance	Estimated parameters of distribution

I (defensive)	I (defensive)	normal	-0.11	1.05	-0.27 ; 0.06	0.95 ; 1.14	μ = -0.11 σ = 1.05
	II (ofensive)	normal	10.2	36.5	-0.2 ; 21.2	32.6 ; 40.4	μ = 10.2 σ = 36.5
II (ofensive)	I (defensive)	normal	-3.26	19.06	-12.72 ; 6,19	14.11 ; 24.36	μ = -3.26 σ = 19.06
	II (ofensive)	normal	0.4	2.9	-0.4 ; 1.4	2.63 ; 3.24	μ = 0.4 σ = 2.9
III (combine)	I (defensive)	normal	3.8	12.8	0.2 ; 10.6	7.8 ; 18.5	μ = 3.8 σ = 12.8
	II (ofensive)	normal	8.4	29.8	-1.4 ; 18.6	26.7 ; 33.2	μ = 8.4 σ = 29.8

Fig.1 *Diagram of histogram of random variable describing a general balance of **A** player on final of the game for his first strategy*

Fig.2 *Diagram of empirical distribution for random variable from **Fig.1***

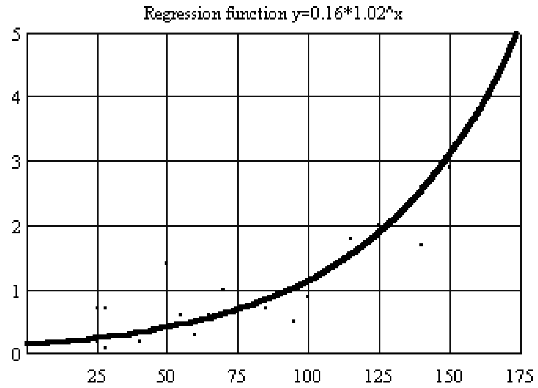


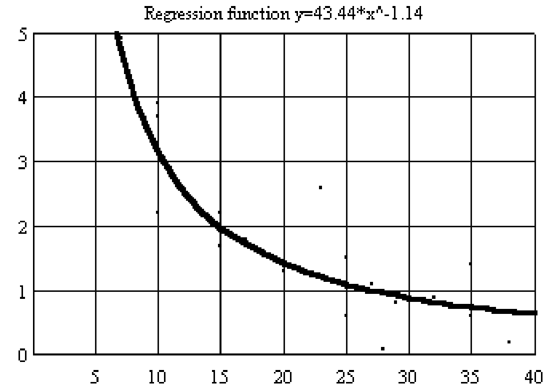


Analysis of regression is presented in ***Fig.3*** and ***Fig.4***. It concern a results of the games for fixed strategy of **A** player (combine stratyegy), fixed scenario of **B** player (defensive scenario) and different initial states of resources for **A** player and fixed initial state of resources for **B** player. Increasing initial state of resources of **A** player (***Fig.3***) his general balance is ascending on his profit. In ***Fig.4*** we show that if time interval between inteligence reports for **A** player is ascending then his general balance on final of the game is descending. This situation have a place because **A** player had more rarely a good information about **B** player and he more often must make a incomplete and uncertain decisions.

Fig.3 *Diagram of regression function describing interdependence between initial state of resources for **A** player and his general balance on final of the game*

Fig. 4 *Diagram of regression function describing interdependence between time interval of inteligence reports and general balance on final of the game for **A** player*





Summary

By reason of limitations volume of this paper we showed only a most important assumptions of the method of evaluation the characteristics of conflict situation which has been modelled by operational game. Estimations of the characteristics defined in section **3** has been above presented. It was the first a goal for us. To realize a second goal (see **Introduction**) we showed a one of methods for players aiding decisions making. One of these methods is using a game theory. In the most simple case we can treate presented situation as a two-person zero-sum game (that means a profits of first player is equal a losses of second one) we can construct a pay-off matrix **M** basing on e.g. mean values of general balance for **A** player presented in ***Tab.2***. We will obtain a following matrix :

$$\mathbf{M} = \begin{array}{c} \begin{bmatrix} -0.11 & 10.2 \\ -3.26 & 0.4 \\ \boldsymbol{3.8} & 8.4 \end{bmatrix} \\ \underbrace{\begin{matrix} \mathrm{I} & \quad\mathrm{II} \end{matrix}}_{\text{Scenarios of } \mathbf{B} \text{ player}} \end{array} \begin{array}{c} \left.\begin{matrix} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{matrix}\right\} \text{Strategies of } \mathbf{A} \text{ player} \\ \\ \end{array}$$

We can notice that value 3.8 (in the first column and the third row) is the bigest in its column and simultanously smallest in its row. This is so-called a game saddle point [5]. We found so-called solution in the set of pure strategies which is following :

- for **A** player - strategy III (combine) ;
- for **B** player - scenario I (defensive).

Strategy III (for **A**) and scenario I (for **B**) are called equilibrium pair of strategies.

However we not always find a solution in the set of pure strategies. In case when not be satisfy situation as above, that is when

$$\sim \underset{\substack{i \in I \\ j \in J}}{\exists} \; \underset{I}{\mathbf{max}} \; m_{ij} = \underset{J}{\mathbf{min}} \; m_{ij},$$

where I describe a set of row numbers and J set of column numbers, we will must find a solution in the set of mixed strategies.

Next step may be adequateness examining of simulation model to simulated reality. In case of game models we can check it knowing a pay-off matrix for players. We can construct the game (in sense of the game theory), solve it well-known methods from the game theory and next we can try to find this matrix in the simulation model. One of the methods for fixing of this matrix may be a method presented above. However, in generality, adequateness examining of simulation model representing such system is enought trouble.

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